Bode Plots

Get to know your logs!

dB	ratio			dB	ratio
-20	0.100			20	10.000
-10	0.316			10	3.162
-5	0.562			5	1.778
-3	0.708			3	1.413
-2	0.794			2	1.259
-1	0.891			1	1.122

- Engineers are very conservative. A "margin" of 3dB is a factor of 2 (power)!
- Knowing a few logs by memory can help you calculate logs of different ratios by employing properties of log. For instance, knowing that the ratio of 2 is 3 dB, what's the ratio of 4?

Bode Plot Overview

- Technique for estimating a complicated transfer function (several poles and zeros) quickly

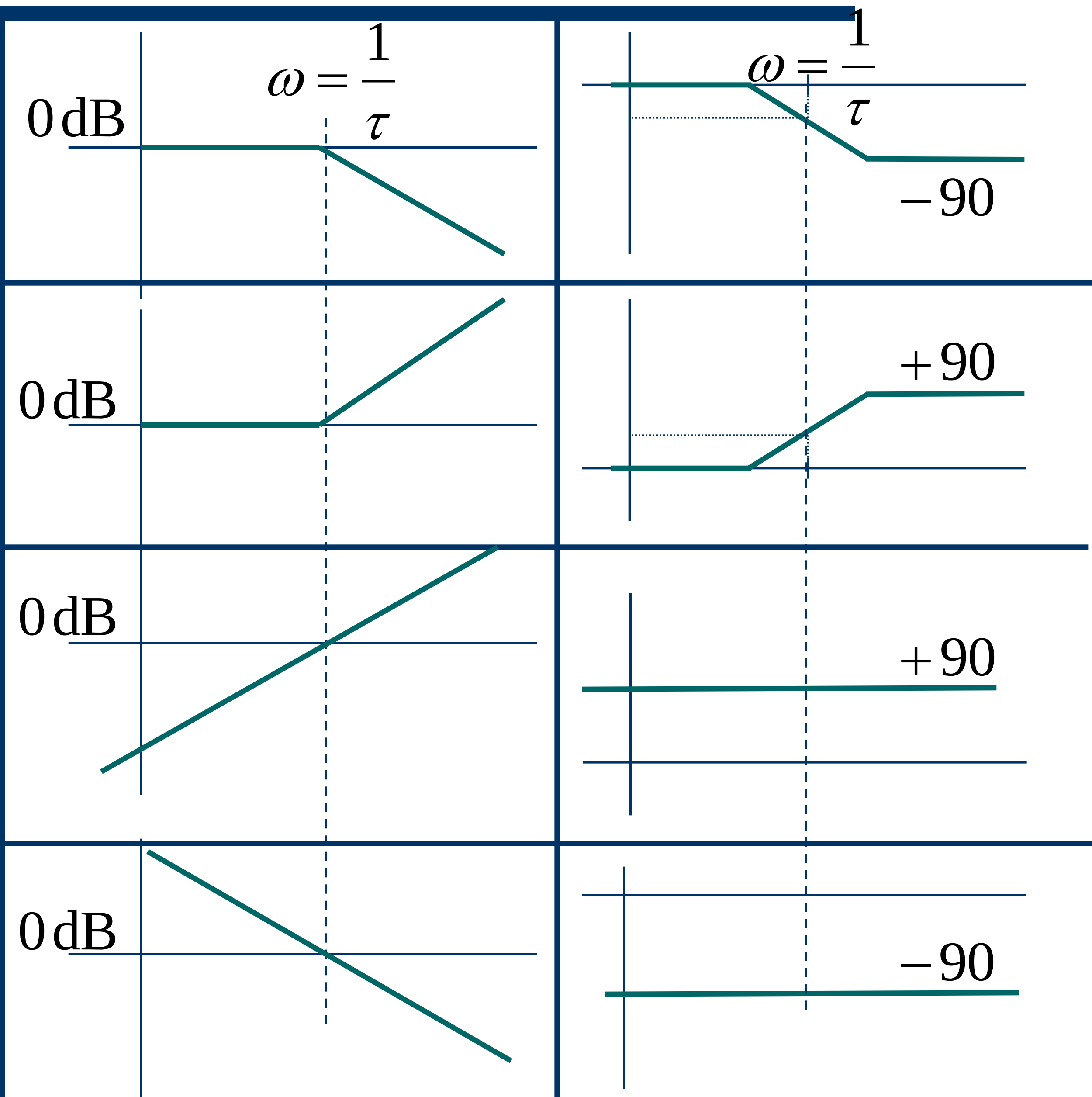
$$H(\omega) = G_0(j\omega)^K \frac{(1+j\omega\tau_{z1})(1+j\omega\tau_{z2})\cdots(1+j\omega\tau_{zn})}{(1+j\omega\tau_{p2})(1+j\omega\tau_{p2})\cdots(1+j\omega\tau_{pm})}$$

- Break frequencies :

$$\omega_i = \frac{1}{\tau_i}$$

Summary of Individual Factors

- Simple Pole: $\dfrac{1}{1+j\omega\tau}$
- Simple Zero: $1+j\omega\tau$
- DC Zero: $j\omega\tau$
- DC Pole: $\dfrac{1}{j\omega\tau}$



Example

- Consider the following transfer function

$$H(j\omega) = \frac{10^{-5} j\omega(1 + j\omega\tau_2)}{(1 + j\omega\tau_1)(1 + j\omega\tau_3)}$$

$$\tau_1 = 100\,\text{ns}$$
$$\tau_2 = 10\,\text{ns}$$
$$\tau_3 = 100\,\text{ps}$$

- Break frequencies: invert time constants

$$\omega_1 = 10\,\text{Mrad/s} \qquad \omega_2 = 100\,\text{Mrad/s} \qquad \omega_3 = 10\,\text{Grad/s}$$

$$H(j\omega) = \frac{\dfrac{j\omega}{10^5}(1 + j\dfrac{\omega}{\omega_2})}{(1 + j\dfrac{\omega}{\omega_1})(1 + j\dfrac{\omega}{\omega_3})}$$

Breaking Down the Magnitude

- Recall log of products is sum of logs

$$\left|H(j\omega)\right|_{\mathrm{dB}} = 20\log\left|\frac{\frac{j\omega}{10^5}\left(1+j\frac{\omega}{\omega_2}\right)}{\left(1+j\frac{\omega}{\omega_1}\right)\left(1+j\frac{\omega}{\omega_3}\right)}\right|$$

$$= 20\log\left|\frac{j\omega}{10^5}\right| + 20\log\left|1+j\frac{\omega}{\omega_2}\right|$$

$$-20\log\left|1+j\frac{\omega}{\omega_1}\right| - 20\log\left|1+j\frac{\omega}{\omega_3}\right|$$

- Let’s plot each factor separately and add them graphically

Breaking Down the Phase

- Since $\angle a \cdot b = \angle a + \angle b$

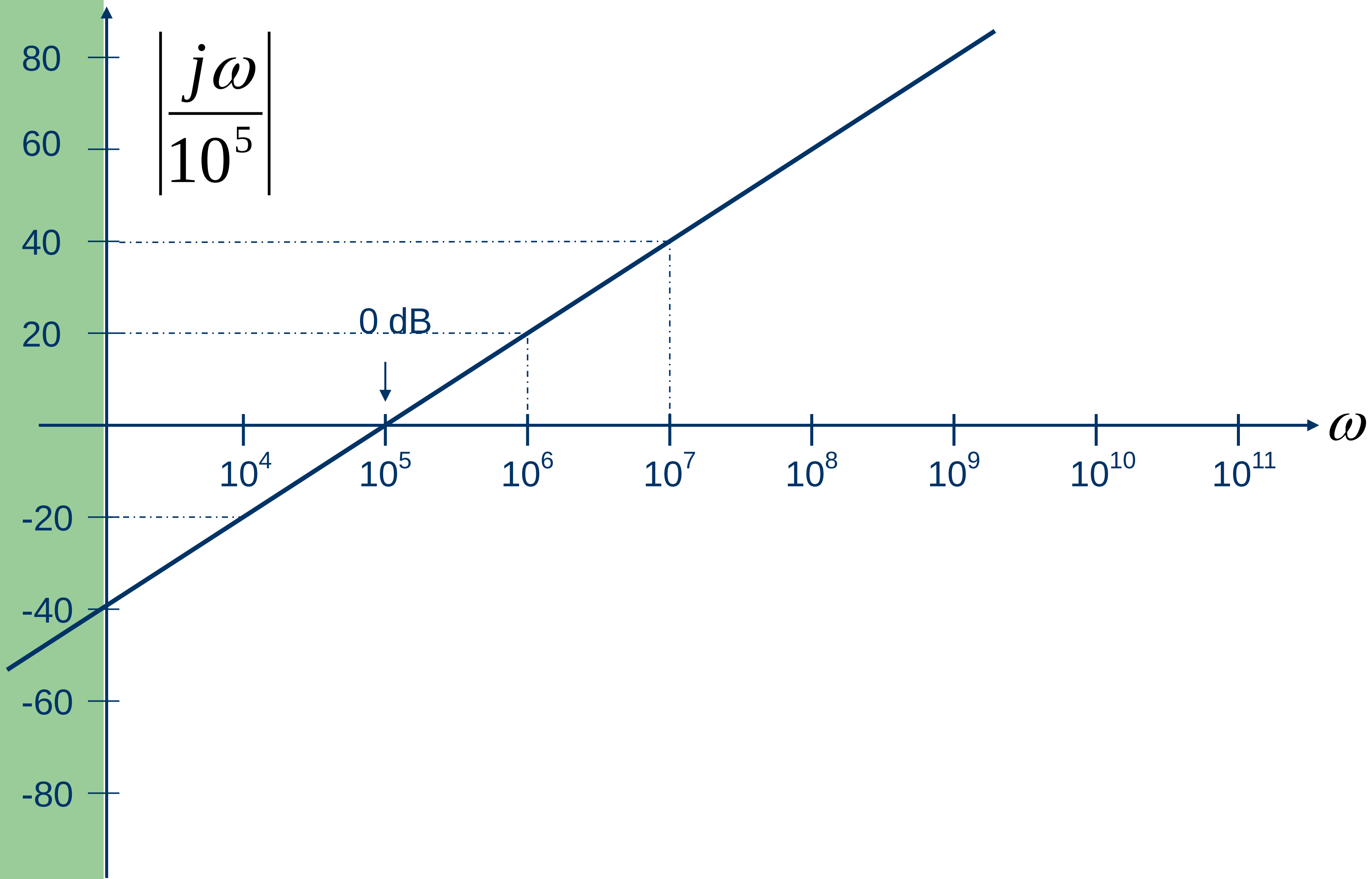
$$\angle H(j\omega) = \angle \frac{10^{-5} j\omega (1 + j\omega\tau_2)}{(1 + j\omega\tau_1)(1 + j\omega\tau_3)}$$

$$\angle H(j\omega) = \angle \frac{j\omega}{10^5} + \angle 1 + j\frac{\omega}{\omega_2}$$

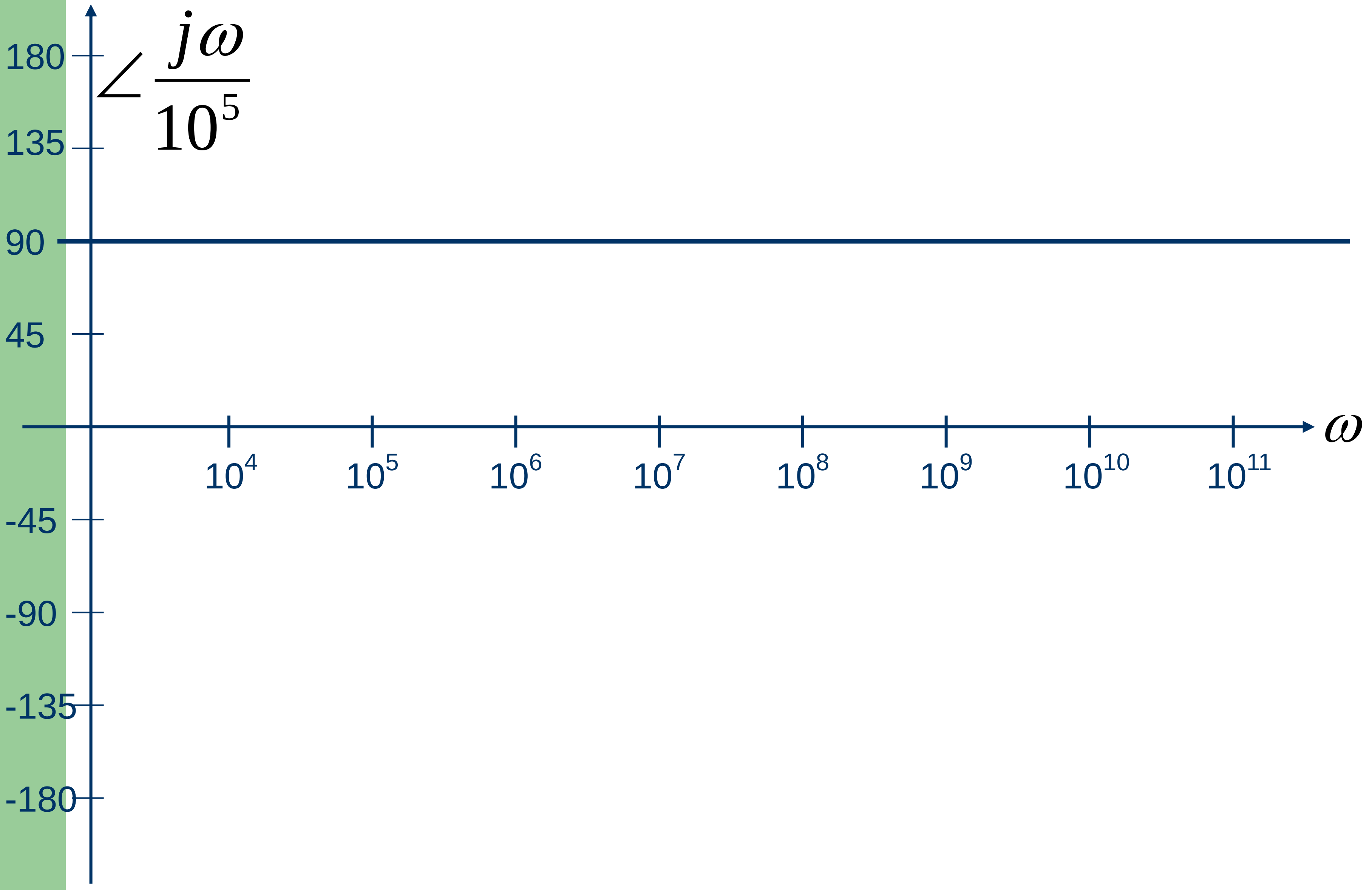
$$- \angle 1 + j\frac{\omega}{\omega_1} - \angle 1 + j\frac{\omega}{\omega_3}$$

- Let’s plot each factor separately and add them graphically

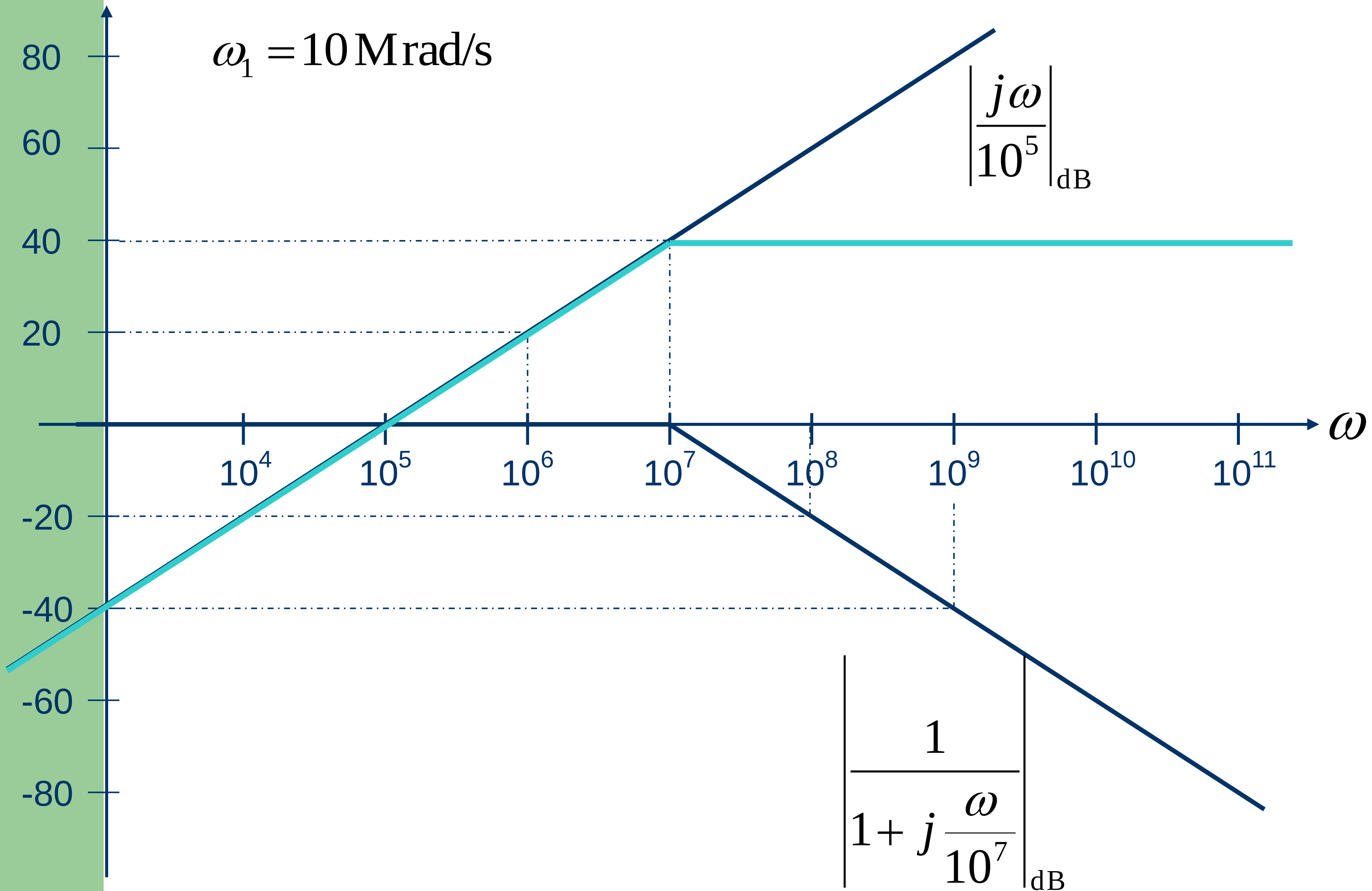
Magnitude Bode Plot: DC Zero



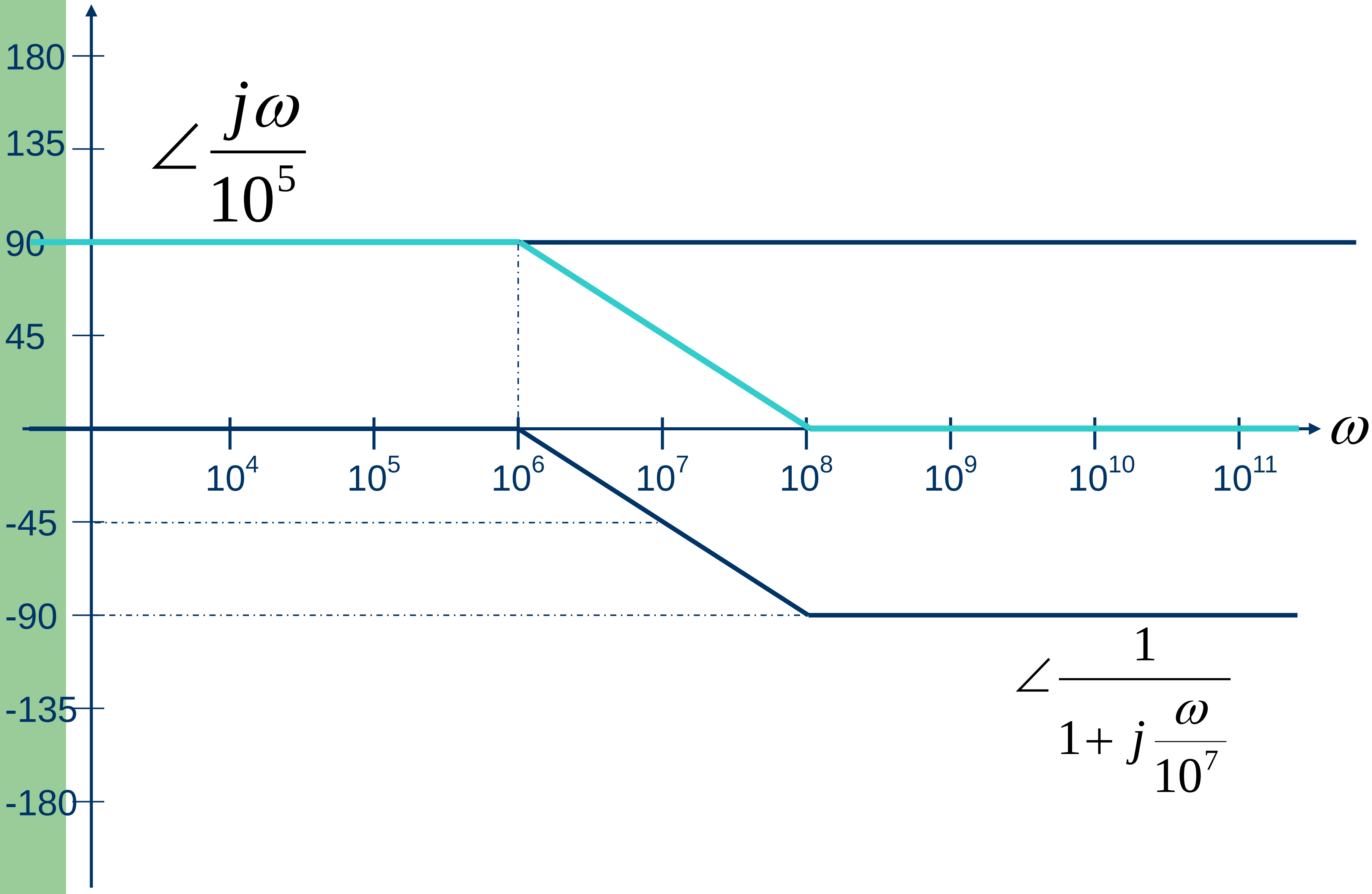
Phase Bode Plot: DC Zero



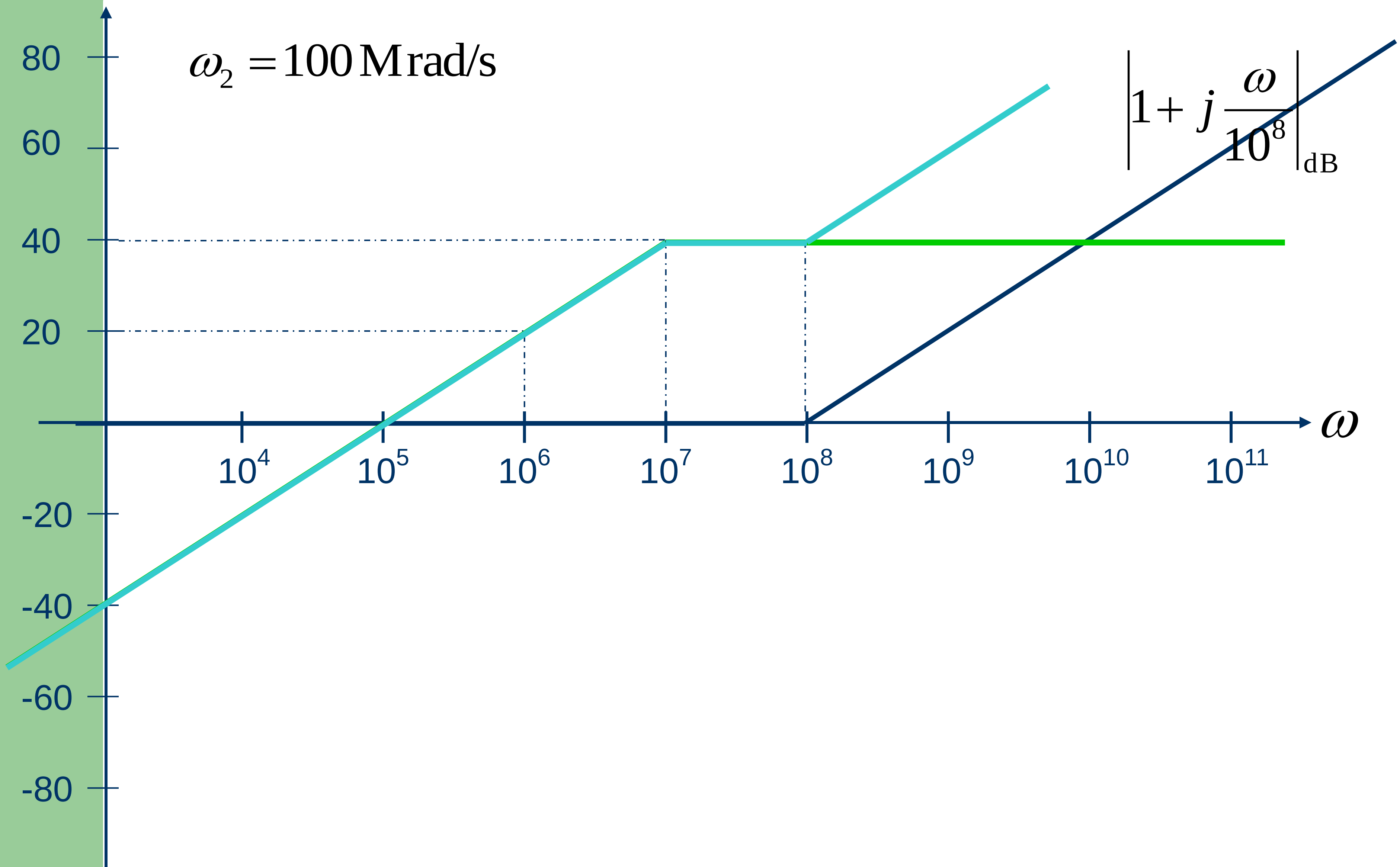
Magnitude Bode Plot: Add First Pole



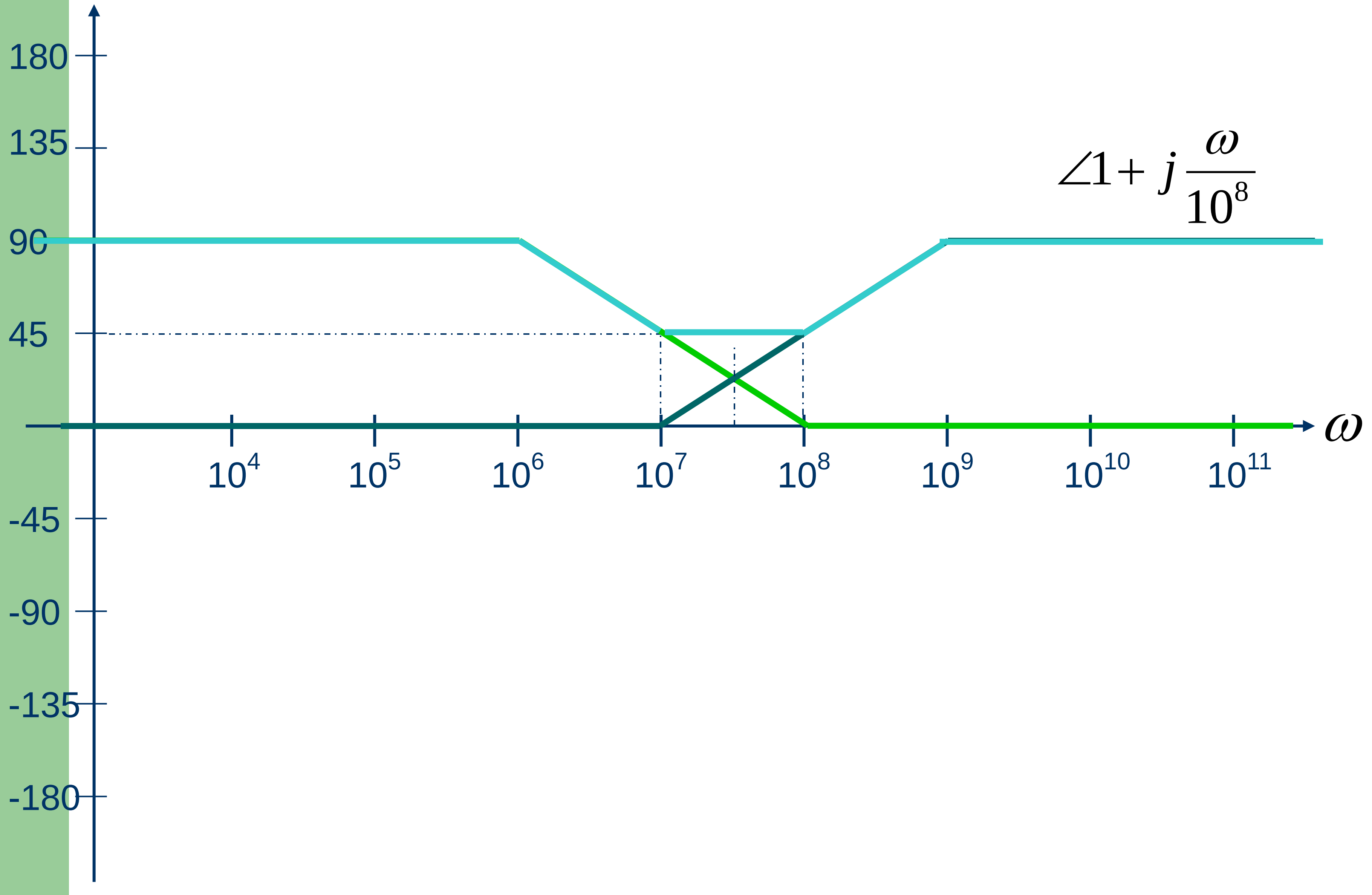
Phase Bode Plot: Add First Pole



Magnitude Bode Plot: Add 2nd Zero



Phase Bode Plot: Add 2nd Zero



Magnitude Bode Plot: Add 2nd Pole